Summary of research projects of Wolfgang Lück (Bonn)
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I have worked and contributed to the following areas:

- Manifold topology
- L^2-invariants
- K-theory
- Transformation groups
- Non-commutative geometry
- Global analysis

Here are some major projects which I have worked on during the last years. Some of them are finished, some are work in progress.

1 Borel Conjecture and Farrell-Jones Conjecture

In the paper [7] with Bartels and Reich we prove the Farrell-Jones Conjecture for algebraic K-theory with coefficient in additive categories for group which are subgroups of finite products of hyperbolic groups or colimits of directed systems of hyperbolic groups. The ingredients are controlled topology and flows on compactifications of a model of the classifying space for proper actions. It has many applications to conjectures by Bass, Moody and Kaplansky. The relevant papers are [7] and [8]. The paper [6] plays an important role in the proof.

The L-theory version is proved in the paper [3] joint with Bartels, where we also deal with CAT(0)-groups. Here also the construction of a geodesic flow for CAT(0)-spaces plays a role, see [5]). This is probably one of my best results of the last five years and is a result of a project I have worked on for the last decade. It implies both the Novikov Conjecture and the Borel Conjecture for such groups in high dimensions. The Borel Conjecture is the topological version of Mostow rigidity. It predicts for a group G that two closed aspherical manifolds M and N whose fundamental groups are isomorphic to G are homeomorphic and that every homotopy equivalence $M \to N$ is homotopic to a homeomorphism.

We can also show (see [1]) that both the Bost Conjecture and the Farrell-Jones Conjecture with coefficients are true for those groups for which Higson-Lafforgue-Skandalis have disproved the Baum-Connes Conjecture with coefficients.

Bartels, Farrell and I prove the Farrell-Jones Conjecture for virtually poly-cyclic groups, cocompact lattices in virtually connected Lie groups and fundamental groups of 3-manifolds (possibly with boundary and possibly non-compact) in [2], where [4] also enters. Bartels-Lück-Reich-Rüping [9] give a proof for arithmetic groups over algebraic number fields, e.g., $GL_n(\mathbb{Z})$. Finally, the case of an arbitrary lattice in a virtually connected Lie group is handled by Kammeyer-Lück-Rüping [30].

I am writing a book on Isomorphism Conjectures in K- and L-theory.

2 Equivariant Chern Characters

In a sequel of papers we develop equivariant Chern characters for equivariant (co-)homology theories for infinite discrete groups and proper G-CW-complexes, see [51, 52, 60, 73]). This is used to compute the source of the assembly map as they appear in the Baum-Connes Conjecture and the Farrell-Jones Conjecture. These methods are used in [60] to compute rationally the topological K-theory of BG for discrete groups G which admit a finite model for the classifying space of proper G-actions.

Maybe the highlight in this project is the paper [51], where an equivariant Chern character for equivariant topological K-theory is constructed and used to show that the Baum-Connes Conjecture implies the (modified) Trace Conjecture. This is already interesting without knowing the Baum-Connes Conjecture because it makes predictions about the L^2-indices of equivariant elliptic complexes over proper cocompact smooth manifolds with invariant Riemannian metric.

3 The Isomorphism Conjecture in K-theory and topological cyclic homology

This projext is joint with Holger Reich, John Rognes and Marco Varisco. Bökstedt, Hsiang and Madsen proved the K-theory version of the Novikov Conjecture for a group using topological cyclic homology, provided the homology of BG is of finite type. In terms of the language of the Isomorphism Conjecture of Farrell and Jones they showed that the assembly map for the family consisting of the trivial subgroup is rationally injective. We generalize this to the family of finite subgroups. This gives a substantially bigger portion in the K-theory of $\mathbb{Z}G$ consisting of the group homology of the Weyl groups $WC = NC/C$ of the finite cyclic subgroups $C \subset G$ and the K-theory of $\mathbb{Z}$, provided the homology of BWC is of finite type for each finite cyclic subgroup $C \subset G$. Roughly speaking, this would prove the Isomorphism Conjecture rationally modulo Nil-terms. The paper [81] is an ingredient in this project. The relevant papers are [80, 79].

4 L^2-Invariants

A book on this topic [54] has appeared in 2002. It contains a detailed presentations of the foundations, main theorems and main open problems about L^2-invariants and includes the (at the time of writing) most recent developments.

My own main contributions are besides some new material in the book [54] above the approximation theorem in [44] and the vanishing theorem for L^2-Betti numbers of mapping tori in [45] which both prove conjectures of Gromov, the computation of L^2-invariant of compact three-manifolds in [41] and [86], and the dimension theory for arbitrary modules over finite von Neumann algebras in [47] and [48].

Here are some recent results:

- **"L^2-torsion and orbit equivalence"**

  Gaboriau has shown that L^2-Betti numbers are invariants under orbit equivalence of groups. Roland Sauer, Christian Wegner and I want to attack the conjecture that the vanishing of the L^2-torsion is an invariant under orbit equivalence if the groups have cocompact models for the classifying space for proper actions and all their L^2-Betti numbers vanish. Partial results can be found in the preprint [85].

- **The limit of $\mathbf{F}_p$-Betti numbers of a tower of finite covers with amenable fundamental groups**

  We prove an analogue of the Approximation Theorem of L^2-Betti numbers by Betti numbers for arbitrary coefficient fields and virtually torsionfree amenable groups. The limit of Betti numbers is identified as the dimension of some module over the Ore localization of the group ring. See [39].

  Bergeron, Linnell, Lück and Sauer [11] study the asymptotic growth of Betti numbers in p-adic analytic towers of covers which is related to results of Calegari and Emerton, and will also present some results for pro-p towers that are not necessarily p-adic analytic.

- **Approximating the first L^2-Betti number of residually finite groups**

  In the paper Lück-Osin [75] we show that the first L^2-Betti number of a finitely generated residually finite group can be estimated from below by using ordinary first Betti numbers of finite index normal subgroups. As an application we construct a finitely generated infinite residually finite torsion group with positive first L^2-Betti number.

  Ershof and Lück [21] give examples, where the $\mathbb{F}_p$-approximation and the $\mathbb{Q}$-approximation for the first Betti numbers of a tower of normal subgroups of finite index with trivial intersection lead to different values for a finitely generated group. This group is not finitely presented. So the question remains open whether we have equality for finitely presented groups.

- **Approximating the L^2-torsion for a tower of finite covers with amenable fundamental groups**

  In [66] we consider the asymptotic behavior of invariants such as Betti numbers, minimal numbers of generators of singular homology, the order of the torsion subgroup of singular homology, and torsion invariants. We will show that all these vanish in the limit if the CW-complex under consideration fibers in a specific way. In particular we will show that all these vanish in the limit if one considers an aspherical closed manifold which admits a non-trivial S^1-action or whose fundamental group contains an infinite normal elementary amenable subgroup. By considering classifying spaces we also get results for groups. This is related to a conjecture of Bergeron and Venkatesh.

  See also [31] for graphs.

- **L^2-torsion functions**

  In a joint project Dubois, Friedl and I investigate the L^2-torsion function which is obtained from L^2-torsion by twisting with a family of representations and is a function $(0, \infty) \to \mathbb{R}$ whose value at 1 is the L^2-torsion. We relate it to the Thurston norm on the first cohomology of a 3-manifold and to the genus of a knot. A survey on the relevant problems is given in [18]. The relevant papers are [19, 16, 17, 26, 67].

- **Universal L^2-torsion, polytopes, L^2-Euler charactistics and applications to 3-manifolds** This is a joint project with Stefan Friedl, see [27, 28, 40] Given an L^2-acyclic connected finite CW-complex, we define its universal L^2-torsion in terms of the chain complex of its universal covering. It takes values in the weak Whitehead group $\mathrm{Wh}^w(G)$. We study its main properties such as homotopy invariance, sum formula, product formula and Poincaré duality. Under certain assumptions, we can specify certain homomorphisms from the weak Whitehead group $\mathrm{Wh}^w(G)$ to abelian groups such as the real numbers or the Grothendieck group of integral polytopes, and the image of the universal L^2-torsion can be identified with many invariants such as the L^2-torsion, the L^2-torsion function, twisted L^2-Euler characteristics and, in the case of a 3-manifold, the dual Thurston norm polytope.

  We also define a twisted version of the L^2-Euler characteristic and study its main properties. In the case of an irreducible orientable 3-manifold with empty or toroidal boundary and infinite fundamental group we identify it with the Thurston norm. We will use the L^2-Euler characteristic to address the problem whether the existence of a map inducing an epimorphism on fundamental groups implies an inequality of the Thurston norms.

5 Analytic and topological torsion and L^2-torsion

This starts with the old paper [43], where I extended the Cheeger-Müller theorem for closed manifolds to closed manifolds with boundary and symmetry. It turns out that the correction term coming from the boundary is very simple, it is essentially the Euler characteristic of the boundary. This paper led to investigation about L^2-torsion. See [41, 46, 54, 69, 82, 84, 86].

6 Boundaries of hyperbolic groups

I have worked with Arthur Bartels and Shmuel Weinberger on the problem when the boundary of a hyperbolic G is a sphere S^n of dimension $n \geq 5$ or has the Čech cohomology of S^n. We recently have finished the proof that the boundary is S^n for $n \geq 5$ if and only if G occurs as the fundamental group of a topological manifold M such that its universal covering $\widetilde{M}$ is homeomorphic to $\mathbb{R}^{n+1}$ and the compactification $\widetilde{M} \cup \partial G$ is homeomorphic to D^{n+1}, see [10].

7 The Burnside Ring and Equivariant Stable Cohomotopy for Infinite Groups

In [57] we propose several definitions for the Burnside ring of an infinite group. We introduce stable equivariant cohomotopy for proper cocompact G-CW-complexes. We formulate a version of the Segal Conjecture for infinite groups in this setting. We have already a promising strategy for its proof and the Atiyah-Segal completion theorem for proper actions of an infinite discrete group has already been proved in [73, 74]. If the Segal Conjecture for infinite groups is true, it seem to be justified to proceed along these lines further and enter new territory. We also want to establish the equivariant stable homotopy category for non-compact Lie groups, having loop groups and Kac-Moody groups in mind. There are interesting analogies between the hypothetical equivariant K-homology of the classifying space for proper group actions for certain loop groups and the hypothetical topological K-theory of the hypothetical group C^*-algebra coming from computations of the first group in terms of representation theory of the loop group. This suggest that there might be a version of the Baum-Connes Conjecture in this setting.

8 Proper equivariant stable homotopy theory

In an ongoing project with Degrijse, Hausmann, Lück Patchkoria and Schwede will establish a good frame work for proper equivariant stable homotopy theory for infinite groups.

9 Computational aspects of the Farrell-Jones Conjecture

Via the conjectures due to Baum-Connes and Farrell-Jones one can try to compute the algebraic K and L-theory of group rings and the topological K-theory of reduced group C^*-algebras by investigating the source of the assembly map. Rational the equivariant Chern characters give some general information, integral computations seem only to be possible in special cases. We have worked out details for hyperbolic groups and certain virtually $\mathbb{Z}^n$-groups, see [15, 83]. In a project joined with Jim Davis we want to apply these to classify manifolds which occur as total spaces of bundles with a torus a fiber and a lens space with cyclic group of prime order as base.

Consider the semi-direct product $\mathbb{Z}^n \rtimes_\rho \mathbb{Z}/m$. A conjecture of Adem-Ge-Pan-Petrosyan predicts that the associated Lyndon-Hochschild-Serre spectral sequence collapses. We prove this conjecture provided that the $\mathbb{Z}/m$-action on $\mathbb{Z}^n$ is free outside the origin. Langer-Lück[36] disprove the conjecture in general, namely, we give an example with $n = 6$ and $m = 4$, where the second differential does not vanish. The topological K-theory of the associated group C^*-algebra is computed in [37] and plays a role the paper of Li-Lück [38], where the K-theory for ring C^*-algebras in the case of higher roots of unity and thereby completely determine the K-theory for ring C^*-algebras attached to rings of integers in arbitrary number fields. By Kirchberg's results this leads also to a full classification of these C^*-algebras.

10 Classing spaces for families

We investigate the notion of classifying spaces for families which is due to tom Dieck and has gotten new attention in connection with the Baum-Connes Conjecture and the Farrell-Jones Conjecture. The following papers deal with the versions, where the family is finite or virtually cyclic [49, 59, 72, 90]. The paper [63] deals with the classifying space for the virtually cyclic subgroups of a group which acts properly cocompactly and isometrically on a CAT(0)-space.

11 Surgery theory

This is one of my early big projects. The relevant papers are [34, 35, 55, 70, 71, 76, 77]. At that time the papers with Madsen [70, 71] had some impact but they address the field of equivariant surgery which is not so active anymore. There we prove a splitting result for the equivariant L-theory computing it in terms of classical L-groups using transfers. If the group under consideration has odd order, this splitting is just a direct sum decomposition since in this situation all the transfer maps are trivial.

Crowley, Macko and I are writing a book on surgery theory.

12 Equivariant Topology of Configuration Spaces and k-regular maps

Blagojević, Lück, Wolfgang and Ziegler [12, 14] study the Fadell–Husseini index of the configuration space $F(\mathbb{R}^d, n)$ with respect to different subgroups of the symmetric group Sym_n. For p prime and $k \geq 1$, we completely determine $\operatorname{Index}^{\mathbb{Z}/p}(F(\mathbb{R}^d, p); \mathbb{F}_p)$. In this process we obtain results of independent interest, including: (1) an extended equivariant Goresky–MacPherson formula, (2) a complete description of the top homology of the partition lattice Π_p as an $\mathbb{F}_p[\mathbb{Z}_p]$-module, and (3) a generalized Dold theorem for elementary abelian groups.

The results on the Fadell–Husseini index yield a new proof of the Nandakumar & Ramana Rao conjecture for a prime. Moreover, for $n = p^k$ a prime power, we compute the Lusternik–Schnirelmann category $\operatorname{cat}(F(\mathbb{R}^d, n)/\operatorname{Sym}_n) = (d-1)(n-1)$, and for spheres obtain the bounds $(d-1)(n-1) \leq \operatorname{cat}(F(S^d, n)/\operatorname{Sym}_n) \leq (d-1)(n-1)+1$.

Blagojević, Cohen, Lück, and Ziegler [13] study k-regular maps $\mathbb{R}^d \to \mathbb{R}^N$, i.e., continuous maps which send any k pairwise disjoint points to k linear independent vectors. We show that N has to exceed a certain expression in terms of d and k if such a map exists.

13 Fibering manifolds

This is a joint work with Tom Farrell and Wolfgang Steimle and the results are described in [23]. The question is when a map between closed manifolds is homotopy equivalent to the projection of a fiber bundle of closed manifolds. This problem was solved by Farrell in the case where the target is S^1. We have figured out the generalization of the torsion obstructions of Farrell to the general case. However, it is obvious that in general the vanishing of this invariants is necessary but not sufficient unless the target is S^1. There exists already the paper [22] about this project. The papers [87] and [88] are relevant for this project.

14 Some smaller projects

- **"Crossed products of irrational rotation algebras by finite groups"**

  This is a joint project with with Siegfried Echterhoff, Chris Phillips and Sam Walters. In [20] we show that the crossed products $A_\theta \rtimes_\alpha F$ and the fixed-point algebras A_θ^F for the action of any finite subgroup $F \subseteq SL_2(\mathbb{Z})$ (which are isomorphic to one of the cyclic groups $\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_6$) on the irrational rotational algebra A_θ via the restriction of the canonical action of $SL_2(\mathbb{Z})$ on A_θ are AF algebras. The same is shown for the flip action of $\mathbb{Z}_2$ on any simple d-dimensional torus A_θ. On the way, we shall prove a number of general results which should have useful applications in other situations.

For this paper computations about the topological K-theory of the reduced group C^*-algebra of certain finite extension of the three-dimensional Heisenberg group were crucial (see [58]).

- **"Topological rigidity of non-aspherical manifolds"**

  We investigate in [33] for which non-aspherical manifolds the Borel-Conjecture or certain variations of it are still true. A closed connected topological manifold M is called *topologically rigid* if any homotopy equivalence $N \to M$ of closed topological manifolds is homotopic to a homeomorphism. The Borel Conjecture for a closed aspherical manifold is equivalent to the statement that M is a topologically rigid. We investigate when a possibly non-aspherical manifold is topologically rigid. We show for a closed connected orientable surface K different from S^2, that $K \times S^d$ is topologically rigid for $d \geq 3$. For product of spheres the answer is related to the Arf-invariant-one-problem. A closed 3-manifold is topologically rigid if and only if its fundamental group is torsionfree. The connected sum of two topologically rigid closed manifolds with torsionfree fundamental group and dimension ≥ 5 is again topologically rigid.

- **"Finiteness obstructions and Euler characteristics of categories"**

  This is a joint project with Roman Sauer and Tom Fiore. We want to generalize the definition of the Euler characteristic due to Leinster and the Möbius inversions for finite categories to EI-categories. We have already a very general K-theoretic Möbius inversion which seems to encompass the standard notions of Möbius inversion in combinatorics. See [25] and [24].

- **"A cocompletion theorem in K-theory for proper actions of a discrete group"**

  This refers to a paper joint with Michael Joachim [29] Let G be a discrete group. We give methods to compute for a generalized (co-)homology theory its values on the Borel construction $EG \times_G X$ of a proper G-CW-complex X satisfying certain finiteness conditions. In particular we give formulas computing the topological K-(co)homology $K_*(BG)$ and $K^*(BG)$ up to finite abelian torsion groups. They apply for instance to arithmetic groups, word hyperbolic groups, mapping class groups and discrete cocompact subgroups of almost connected Lie groups. For finite groups G these formulas are sharp. The main new tools we use for the K-theory calculation are a Cocompletion Theorem and Equivariant Universal Coefficient Theorems which are of independent interest. In the case where G is a finite group these theorems reduce to well-known results of Greenlees and Bökstedt. The cocompletion Theorem is obtained by dualizing the equivariant version of the Atiyah-Segal completion theorem for proper actions of an infinite discrete group G in [73, 74].

- **"Equivariant principal bundles and their classifying spaces"**

  In the paper [89] Uribe and I consider Γ-equivariant principal G-bundles over proper Γ-CW-complexes with prescribed family of local representations. We construct and analyze their classifying spaces for locally compact, second countable topological groups with finite covering dimension Γ and G such that G almost connected.

15 Survey articles

We mention the following survey articles: [18, 50, 51, 53, 54, 59, 61, 62, 65, 64, 68, 69, 78].

16 Books

We mention the following books [32, 42, 54, 56].

Crowley, Macko and I are writing a book on surgery theory. I am writing a book on Isomorphism Conjectures in K- and L-theory.

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