Optimal delegation in common agency

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Abstract

In this paper, I use a generalized cheap talk framework (Crawford-Sobel) to consider the delegation problem with two principal and one agent. I first consider the benchmark of cooperative and non-cooperative situation under complete information. Then consider the cooperative situation under incomplete information.In this situation, the problem boils down to the optimal control problem with control instrument in two dimentions. Finally, I consider the delegation under common agency problem. The priliminary result shows that when the sender can only send public message, due to the stategic interation between the principals, the delegation problem collapses to a communication problem without commitment power of the principals. When the sender can send private messages to the principals, it is equivalent to two independent delegation problems considered by Alonso and Matouschek 2005. Whether sending the public message is better than sending the private message depends on the parameter value.

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1 Introduction

There are many situations in which several principals with diverse interests want to elicit information from the same agent. The voters and super leader may want to know some officer's announcement on his objectives. Different shareholders with different policy preference may want to know the demand for different products from the manager. In all these settings, each principal want to choose the optimal delegation rule to elicit information from the biased agent, taking into accout the message sending by the agent is the best response to all the delegation rules of all the principals. In other words, there is some externality between the principals brought by the interactions with the agent.

In this paper, I consider a situation where two uninformed principles simultaneously try to elicit the information of the true state from the informed agent. There are many situations where those holding formal power to make decisions lack critical information or expertise. Often, this information or expertise lies in the hands of subordinates having conflicts of interest over the ultimate decision. An important objective in the design of institutions is to facilitate the communication of private information. There are two major potential roles for the communication of private information. The first, a planning role, is to improve an uninformed party's decision choices through elicitation of relevant private information. The second, a control role, particularly applicable in agency relationships, is to influence agents' unobservable action choices. Research on the roles communication plays in various relationships has focused on settings differing along two prominent dimensions: whether transfers between the uninformed party and the informed party are feasible and whether the uninformed party can credibly commit to his decision rule. Here, we consider the situation where two principals can credibly commit to their decision rules and there does not exist transfers between them. In this case, when one principal optimize his decision rule, he must take into account what is the other principal's rule. My preliminary result shows that when the agent can only send the public message, due to the stategic interation between the principals, the delegation problem collapses to a communication problem without commitment power of the principals.

The paper is organized as follows: In section 3 I begin to describe the model. Then I consider the benchmark model under complete information in section (3.1). In section 3.2, I consider the cooperative solution to the delegation problem under incomplete information. And in the section 3.4, I extend to the case under non-cooperation and incomplete information.

2 Literature Review

Within the framework of Crawford and Sobel, the literature on delegation have been widely studied. Holmstrom (1977,1984) considers a general version of the set up and proves the existence of an optimal delegation set or, equivalently, an optimal desision rule. He then characterizes optimal interval delegation sets, i.e. delegation sets in which the agent can choose any decision from a specific interval; Melumad and Shibano (1991) characterize the optimum among all compact delegation sets; Dessein (2002) considers a model in which a principal can either let an agent make any decision or engage in cheap talk communication and then make the decision herself. Martimort and Semenov (2006) consider a setting with multiple agents and provide a sufficient condition for threshold delegation to be optimal; Alonso and Matouschek (2007) consider the delegation problem under partial commitment and full commitment by the principal and endogenized the commitment power in delegation problem by using a dynamic model where the principal and the agent repeatedly interact. Martimort and Semenov (2008) have extended this mechanism design approach to the case of multiple privately informed agents dealing with a single principal in a political economy context. Martimort and Koessler (2009)extend the analysis to the design of multi-dimensional communication mechanisms.Wolinsky (2002) consider a mutiple agents model where the DM want to elicit information from the agents, he consider both cases in which the principal can commit to a mechanism and can not commit to it. In this paper, I try to extend this strand of literature into multiple principle and one privately informed agent situation. Although Kovac and Mylovanov (2006) show that the optimal mechanism may not be deterministic in a setting with a continuum of states and decisions, I here only consider the deterministic delegation rules due to the simplicity.

Another strand of literature this paper is related to is common agency. This paper may be seen as an extension of common agency problem (Bernheim and Whinston 1986) to settings without transfer. Stole (1991) extend the common agency problems in a context of adverse selection. Martimort (1996) compare the choice between common agency and competing manufacturer-retailer hierarchies. Martimort (2007) gives a excellent survey on common agency literature. The main concern I think in the common agency paper is the stategic interacition through the incentive contracts given to the agent. There exist some externality on one contract to the other due to the distortion in one contract may do good or harm to the other. Here, we also have this kind of externality that one principal may take advantage of the information potentially disclosed by the other contract. We can also view this paper as common agency problem without transfer.

3 Model

There are three players, one sender and two receives. The sender observes the state of the world $\theta \in \Theta = [0,1]$, while the receivers do not observeθ . The common prior over the states of the world is a continuous distribution F on Θ. Each receiver i can choose an action $y_i \in R$.

The timing is as follows. First the principals "Promise" to make her decision according to a decision rule $a(m) : M \to A$ that maps the agent's message space M into projects. Second, the agent learns the state θ and sends a costless message $m \in M$. We assume that $M = Y$ and we say that the agent "recommends" a project y if he sends a message $m = y$. Third, the principal decides what project to implement. We say that the principal "rubber-stamps" the agent's recommendation if, in response to receiving the message $m = y$, she implements project y. The principals and the agent realize $U_{P_1}(y_1(m), \theta, b_1)$, $U_{P_2}(y_2(m), \theta, b_2)$ and $U_A(y_1(m), y_2(m), \theta)$ respectively.

We assume that the utility function of sender i is $u(y_1, y_2, \theta) = -l_1(|y_1 - \theta|) - l_2(|y_2 - \theta|)$, $l_i^{'} > 0, l_i^{''} > 0, \forall x > 0$, and the utility function of receiver i is $v_i = -L(|y_i - \theta - b_i|)$, $L^{'}(x) > 0, L^{''}(x) > 0, \forall x > 0$, where $b_i \in R$. Given these preferences,the sender's most preferred actions in state θ are $y_1 = y_2 = \theta$; receiver i's most preferred action is $y_i = \theta + b_i$. The utility of each party in state θ decreases in the distance from the preferred action(s) given θ to the action(s) that is(are) actually taken. A special case are the quadratic preferences ($l_1(x) = l_2(x) = L(x) = x^2$), which are assumed in many applications.

Information: The agent learns the realization of the state θ but the principal does not. It is commonly known, however, that θ is drawn from a cumulative distribution function $F(\theta)$. The corresponding probability density function $f(\theta)$ is absolutely continuous and strictly positive for all $\theta \in \Theta$.

3.1 The complete information

Here, we assume the utility of the two principles is addictive. Under complete information and cooperation between the principals, the total utility of the the coalescence is $U_P(y(m), \theta) = -(y_1 - \theta - b_1)^2 - (y_2 - \theta - b_2)^2$. The optimal action can be deduced easily: $y_1 = \theta + b_1$, $y_2 = \theta + b_2$

With complete information and in the absence of a cooperative behavior between the principals, it is easy to check that the same action as above will be chosen. Because the each principle has the decision power over the actions and the information is perfect.

3.2 The incomplete information and Cooperation

Formally, an equilibrium of the stage game is characterized by (*i.*) the agent's communicatin rule $\mu(\theta) : \Theta \to \Delta M$ which specifies the probability of sending message $m \in M$ conditional on observing state θ, (*ii.*) the principal's decision rule $y(m) : M \to Y$ which maps messages into projects and (*iii.*) the principal's belief function $g(\theta|M) : M \to \Delta\theta$ which states the probability of state θ conditional on oberving message m. In a Perfect Bayesian Equilibrium the communication rule is optimal for the agent given the decision rule, the decision rule is optimal for the principal given the belief function and the belief function is derived from the communication rule using Bayes' rule whenever possible.

The decision outcome function is said to be attainable if there exists a decision mechanism which yields $y[m(\theta)] = y(\theta)$ at a Nash equilibrium.

Reveleation Principle: An delegation outcome function $y(.)$ is attainable if and only if the decision mechanism $(y(.), \Theta)$ has a Nash equilibrium such that $m(\theta) = \theta$

This principle shows that the principal need not look at general message spaces M in the design problem. He can always take $M = Y$ and concentrate on the choice of the decision function alone. However, he can only consider decision functions that yield truth-telling Nash equilibria.

The key tradeoff that the principle faces when she considers the many different organizational arrangements is between the direct cost of biasing her decisions in favor of the agent and the indirect benefit of inducing the agent to reveal more information.

The optimal delegation scheme $(y^*(m)), \mu^*(\theta)$ that maximizes the principal's expected payoff therefore solves

$$\max_{y(m),\mu(\theta)} E_\theta[U_P(y(m), \theta)] \tag{1}$$

subject to the agent's incentive compatibility constraint

$$\mu(\theta) \in arg \max_{m \in M} U_A(y(m), \theta) \tag{2}$$

Where $U_P(y(m), \theta) = -(y_1 - \theta - b_1)^2 - (y_2 - \theta - b_2)^2$, $U_A(y(m), \theta) = -(y_1 - \theta)^2 - (y_2 - \theta)^2$

So the problems can be rewritten as,

$$\max_{y_1(m(\theta)),y_2(m(\theta))} \int_{\underline{\theta}}^{\overline{\theta}} [-(y_1(m)-\theta-b_1)^2-(y_2(m)-\theta-b_2)^2]f(\theta)d\theta \tag{3}$$

$$S.t. \;\; m(\theta) \in arg\max_{m(\theta)} -(y_1(m)-\theta)^2-(y_2(m)-\theta)^2 \tag{4}$$

Applying revelation principal, we can further rewrite it as,

$$\max_{y_1(\theta),y_2(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [-(y_1(\theta)-\theta-b_1)^2-(y_2(\theta)-\theta-b_2)^2]f(\theta)d\theta \tag{5}$$

$$S.t. \;\; \theta \in arg\max_{\theta'} -(y_1(\theta')-\theta)^2-(y_2(\theta')-\theta)^2 \tag{6}$$

We can use first-order condition to replace the incentive constraint (6) by

$$y_1'(\theta)(y_1(\theta)-\theta)+y_2'(\theta)(y_2(\theta)-\theta)=0 \tag{7}$$

Lemma 1:The necessary and sufficient condition for incentive compatibility is that $(y_1(\theta)+y_2(\theta))$ is non-decreasing in θ and thus a.e. differentiable in θ. At any differentiability point, we have:

$$y_1'(\theta)+y_2'(\theta) \geq 0 \tag{8}$$

$$y_1'(\theta)(y_1(\theta)-\theta)+y_2'(\theta)(y_2(\theta)-\theta)=0 \tag{9}$$

Proof: Incentive compatilibty implies for all pairs $(\theta,\widehat{\theta}) \in \Theta^2$:

$$\sum_{i=1}^{2}(y_i(\widehat{\theta})-\theta)^2 \geq \sum_{i=1}^{2}(y_i(\theta)-\theta)^2 \tag{10}$$

$$\sum_{i=1}^{2}(y_i(\theta)-\widehat{\theta})^2 \geq \sum_{i=1}^{2}(y_i(\widehat{\theta})-\widehat{\theta})^2 \tag{11}$$

Summing those inequalities yields:

$$\sum_{i=1}^{2}(y_i(\theta)-y_i(\widehat{\theta}))(\theta-\widehat{\theta}) \geq 0 \tag{12}$$

Thus, $\sum_{i=1}^{2} y_i(\theta)$is non-decreasing in θ. Therefore, it is almost everywhere differentiable with, at any differtiability point, a derivative such that (8) holds. At such a point, an incentive compatible mechanism must also satisfy the first-order condition of the agent's revelation problem, namely (9).

To give a complete characterization of the optimal control problem, we need some recent result of the Caculus of variation (Clarke,1990) like Martimort and Koessler (2009).

Definition 1 (Centralization). Under centralization the only project the principal rubber-stamps if $y = E(\theta)$, i.e. her preferred project given her prior beliefs. If the agent recommends any other project she overrules him and implements $y = E(\theta)$.

Given this decision making by the principal, it is optimal for the agent to always recommend $y = E[\theta]$ The agent's information is therefore not used under this delegation scheme.

Definition 2 (Threshold Delegation). Under threshold delegation the principal rubber-stamps any recommendation below a threshold project $(y_1 + b)$ and she overrules the agent and implements $(y_1 + b)$ if he recommends a project above the threshold.

Definition 3 (Menu Delegation). Under menu delegation the principal offers a menu with a finite number of projects and rubber-stamps any project ont the menu. If the agent recommends a project that is not on the menu, the principal overrules him and implements one of the projects that is on the menu.

Under menu delegation, therefore, the agent can choose between a finite number of projects.

The problem considered here is very similar to Alonso and Matouschek (2007) except that now our objective functions of the principals and agent are different in the sense that now the principal has two dimentional instrument to use for screening. we first recall some basic result of Alonso and Matouschek (2007) in the one dimentional case:

Lemma 1. Suppose $G(\theta) \equiv F(\theta) + bf(\theta)$ is strictly increasing in θ for all $\theta \in \Theta$. Then threshold delegation is optimal. If $G(\theta) \equiv F(\theta) + bf(\theta)$ is stictly decreasing in θ for all $\theta \in \Theta$, then centralization is optimal.

In the two dimentional case, I can duduce that the two principals can do better than under separately delegation solution suggested by Alonso and Matouschek (2007).

3.3 The incomplete information and Non-Cooperation

Let us consider now the case of a noncooperative behavior between the two principals under incomplete information. Here, we consider the case that after observing the state θ the agent sends a message m that is observed by both principals, in other words, we only consider public communications. Each principal P_i maximizes his uitility taking into account the effect that his delegation rule has on the choice of both $y_1(m)$ and $y_2(m)$. Now the utility of the agent becomes:

$$U_A(y_1(m), y_2(m), \theta) = -(y_1(m) - \theta)^2 - (y_2(m) - \theta)^2$$

For the ease of analysis, let us assume $l_1 = l_2$ afterwards. We can write P_1's problem under incomplete information as

$$\max_{y_1(m), m(\theta)} E_\theta[U_P(y_1(m), \theta)]$$

subject to the agent's incentive compatibility constraint

$$m(\theta) \in arg \max_{m \in M} U_A(y_1(m), y_2(m), \theta)$$

Lemma 2: Threshold delegation is never an equilibrium.

Proof: to get the intuiation, assume $b_1 = b_2$ without loss of generality. Suppose there exit some interval $\theta \in (\theta_1, \theta_2)$ in which $y_1(m = y) = y$. The best response of principal 2 to the delegation rule of principal 1 in this interval is $y_2(m = y) = y - 2b_2$. Under the two rules the agent will send a message $m = y + b_2$ which is exactly what the principal 2 wants. Again, the best response of principal 1 to the rule of principal 2 is $y_1(m = y) = y - 2b_2 - 2b_1$. So on and so forth, we can easily see there is no pure strategy Nash equilibrium.

So the only possibility is using the interval delegation. At the cutoff point of different intervals, the agent must be indifferent between the adjacent interval actions of the two principals. So the partition of the state space must be the same for the two principals at equilibrium. In case where $b_1 \neq b_2$, the actions chosen by the principals must be different. I also claim that the stategic reaction of one principal to the other's delegation rule will completely destroy the value of commitment power which are usually positive in the one principal case. In other words, the two principals are playing the generalized Crowford-Sobel's cheap talk game at equilibrium. In this public message sending situation, the potential value of commitment power is totally destroyed by the free-riding problem between the two principals.

Lemma 3: In every equilibrium of the public communication game after any equilibrium message the actions of the receivers are related as follows,

$$y_2 - y_1 = b_2 - b_1$$

The result follows from the fact that message m is publicly observed by both receives, and thus they have identical posterior distrubutions of the states.

Given these two lemma, we can characterize the equilibria of the public information delegation game.

Proposition 2: In the two receiver one sender delegation game, the optimal delegation rule is menu delegation. Let $\frac{b_1+b_2}{2} \neq 0$.

(i) Any equilibrium of the public communication game is characterized by a sequence of cutoff type $0 = \theta_0 < \theta_1 < .... < \theta_N = 1$ such that the equilibrium outcome $y(\theta)$ is a constant action pair on every interval (θ_k, θ_{k+1}) for $i = 1, 2$

(ii) There is an equilibrium of the public communication game characterized by cutoff types $0 = \theta_0 < \theta_1 < .... < \theta_N = 1$ if and only if the CS game between the sender with utility function $-(y-\theta)^2$ and the receiver with utility function $-(y-\theta-\frac{b_1+b_2}{2})$ has an equilibrium with the same cutoff types.

In the above, we just assume that the message is public. If we allow the private message sending, we can just get the result by Alonso and Matouschek 2005.

3.3.1 Comparison between public message sending and private message sending

From the principal's point of view, whether public message communication is better than private message sending depends on the parameter value b_1, b_2. For example, if $b_1 = -b_2$, then in the equilibrium of public message sending game the sender will exactly disclose the true state θ, and the principals just implement the best action $y_1 = \theta + b_1, y_2 = \theta + b_2$ which is better than the actions under private message communication.

There are also some values on b_1, b_2 under which private message sending is better. For example, when b_1 is very small but b_2 is so large that $\frac{b_1+b_2}{2}$ is large enough to guarantee the only equilibrium is babbling. Yet since b_1 is very small, there is some gains from delegation if the sender can privatly communicate with principal 1.

3.4 Concluding remarks

My objective of this paper is try to find the optimal delegation rule under common agency. When the sender can only send the public message, the externality between the two principals is so severe that the threshold delegation is never optimal. The only possible optimal delegation rules are interval delegation where there exist some noisy information transmisson and the centralization where no information is disclosed. In other words, the stategic reaction of one principal to the other's delegation rule will completely destroy the value of commitment power which are usually positive in the one principal case. In other words, the two principals are playing the generalized Crowford-Sobel's cheap talk game at equilibrium. In this public message sending situation, the potential value of commitment power is totally destroyed by the free-riding problem between the two principals.

My conculsion depends on the natural extension of the framework of Crawford and Sobel (1982) in which we assume that the payoffs of each receiver are independent of the action of the other receiver, and that the sender's payoff is separable in the action of the two receivers.

This paper does not claim that the cheap-talk equilibrium is the only equlibrium of this delegation game. I need to work out whether there exist other equilibriums. As pointed out by Martimort and Stole (2002), I may put many restrictions to consider only the pure strategy equilibria and a single mechanism. This leaves some room for more complex mechanism, for example, stochastic mechanism and menus of mechanism which may improve the current equilibrium.

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