

Bounds for the Cubic Weyl Sum

Roger Heath-Brown

Abstract

Subject to the *abc*-conjecture, we improve the standard Weyl estimate for cubic exponential sums in which the argument is a quadratic irrational. Specifically we show that

$$\sum_{n \leq N} e(\alpha n^3) \ll_{\epsilon, \alpha} N^{5/7+\epsilon}$$

for any $\epsilon > 0$ and any quadratic irrational $\alpha \in \mathbb{R} - \mathbb{Q}$. Classically one would have had the (unconditional) exponent $3/4 + \epsilon$ for such α .