Base Change for Hilbert Eigenvarieties of Unitary Groups

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Abstract

Coleman [C1], [C2], continuing works of Serre, Katz, Dwork, Hida, Gouvea-Mazur, showed that each eigenform of level $\Gamma_1(p)$ at a fixed prime p belongs to a p-adic family of eigenforms. Thus he constructed an analytic family of overconvergent modular forms and applied to the operator U_p the spectral theory of compact operators of orthonormalizable Banach modules, which he introduced. These ideas were developed by Coleman and Mazur [CM] to construct a p-adic analytic curve, which they named the eigencurve, parametrizing the overconvergent modular forms of finite slope and fixed tame ramification. The existence of p-adic families of cusp eigenforms ordinary at p had been studied by Hida. Buzzard [B] showed that the construction of families in the case of quaternion algebras over $\mathbb{Q}$ which are ramified at infinity can be dealt with directly by elementary means, also in the Hilbert modular case, namely over a totally real field F instead of $\mathbb{Q}$. Chenevier [Ch1] developed the theory to apply to unitary groups over $\mathbb{Q}$ which are compact at the real place. The compactness at infinity eliminates many geometric difficulties, in fact $G(\mathbb{Q})\backslash G(\mathbb{A})/U$ is finite for any open compact subgroup U of $\mathbb{Q}(A)$, and so all automorphic forms on contribute only to cohomology in degree zero. Following Chenevier closely, we extend his theory to the Hilbert case, that is to unitary groups over a totally real field F which are anisotropic at each archimedean place. This permits us to relate the eigenvarieties which we construct for two totally real fields, one being a cyclic extension of the other. Thus we construct a base change morphism from one eigenvariety to the other, using the basechange lifting for automorphic representations of these two groups