

# Minimizers of the Willmore functional under fixed conformal class

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**Abstract:** We prove the existence of a smooth minimizer of the Willmore energy in the class of conformal immersions of a given closed Riemann surface into  $\mathbb{R}^n$ ,  $n = 3, 4$ , if there is one conformal immersion with Willmore energy smaller than a certain bound  $\mathcal{W}_{n,p}$  depending on codimension and genus  $p$  of the Riemann surface. For tori in codimension 1, we know  $\mathcal{W}_{3,1} = 8\pi$ .

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